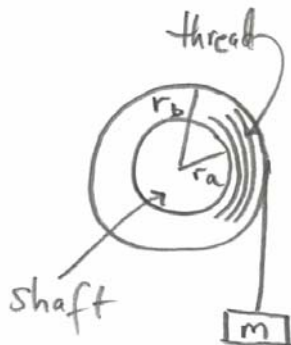


Problem 3: Stall Torque of Motor

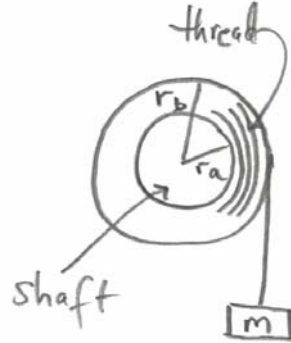
The following simple experiment can measure the stall torque of a motor. (See sketch.) A mass m is attached to one end of a thread. The other end of the thread is attached to the motor shaft so that when the motor turns, the thread will wind around the motor shaft. The motor shaft without thread has radius $r_0 = 1.2 \times 10^{-3} \text{ m}$. Assume the thread winds evenly effectively increasing the radius of the shaft. Eventually the motor will stall.



- a) Suppose a mass $m = 5.0 \times 10^{-2} \text{ kg}$ stalls the motor when the wound thread has an outer radius of $r_f = 2.4 \times 10^{-3} \text{ m}$. Calculate the stall torque.
- b) Suppose the motor has an unloaded full throttle angular frequency of $\omega_0 = 2\pi f_0 = 2\pi(6.0 \times 10^1 \text{ Hz})$ (unloaded means that the motor is not applying any torque). Suppose the relation between angular frequency ω and the applied torque τ of the motor is given by the relation
- $$\omega = \omega_0 - b\tau$$
- where b is a constant with units $\text{s/kg} - \text{m}^2$. Using your result from part a), calculate the constant b . Make a graph of ω vs. τ .
- c) Graph the power output of the motor vs. angular frequency ω . At what angular frequency is the power maximum? What is the power output at that maximum? Briefly explain the shape of your graph. In particular, explain the power output at the extremes $\tau = 0$ and $\tau = \tau_{\text{stall}}$.
- d) What torque does the motor put out at its maximum power output?

Problem 3: Stall Torque of Motor

The following simple experiment can measure the stall torque of a motor. (See sketch.) A mass m is attached to one end of a thread. The other end of the thread is attached to the motor shaft so that when the motor turns, the thread will wind around the motor shaft. The motor shaft without thread has radius $r_0 = 1.2 \times 10^{-3} \text{ m}$. Assume the thread winds evenly effectively increasing the radius of the shaft. Eventually the motor will stall.



- a) Suppose a mass $m = 5.0 \times 10^{-2} \text{ kg}$ stalls the motor when the wound thread has an outer radius of $r_f = 2.4 \times 10^{-3} \text{ m}$. Calculate the stall torque.

Answer:

The torque is $\tau_{st} = r_f mg \approx 1.1 \times 10^{-3} \text{ N} \times \text{m}$.

- b) Suppose the motor has an unloaded full throttle angular frequency of $\omega_0 = 2\pi f_0 = 2\pi(6.0 \times 10^1 \text{ Hz})$ (unloaded means that the motor is not applying any torque). Suppose the relation between angular frequency ω and the applied torque τ of the motor is given by the relation

$$\omega = \omega_0 - b\tau$$

where b is a constant with units $\text{s/kg} \cdot \text{m}^2$. Using your result from part a), calculate the constant b . Make a graph of ω vs. τ .

Answer:

When the motor is stalled, $\omega = 0$. Thus $\omega_0 = b\tau_{st}$ and

$$b = \frac{\omega_0}{\tau_{st}} = \frac{\omega_0}{r_f mg} \approx 3.2 \times 10^5 \frac{s}{kg \times m^2}$$

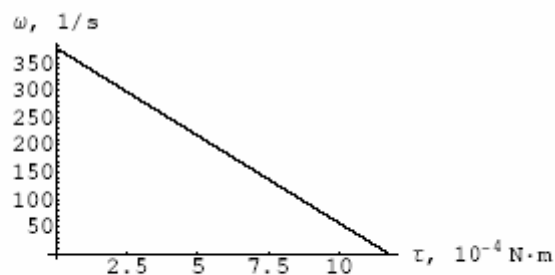


Figure 1: Angular velocity vs. Torque

- c) Graph the power output of the motor vs. angular frequency ω . At what angular frequency is the power maximum? What is the power output at that maximum? Briefly explain the shape of your graph. In particular, explain the power output at the extremes $\tau = 0$ and $\tau = \tau_{stall}$.

Answer:

- c) The power output is given by

$$P = \tau\omega = \frac{\omega_0 - \omega}{b}\omega.$$

The plot is a parabola (Fig. 2)

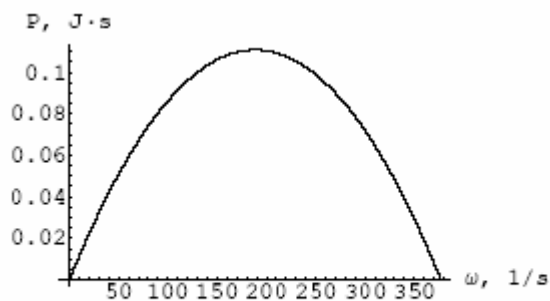


Figure 2: Power output vs. angular velocity

- d) What torque does the motor put out at its maximum power output?

Answer:

d) The maximum is reached at $\omega = \omega_0/2$. The corresponding output power is $\omega_0^2/(4b) \approx 0.1W$.

Problem 4 (Conservation of Angular Momentum)

A meteor of mass $m = 2.1 \times 10^{13} \text{ kg}$ is approaching earth as shown on the sketch. The radius of the earth is $r_e = 6.37 \times 10^6 \text{ m}$. The mass of the earth is $m_e = 5.98 \times 10^{24} \text{ kg}$. Suppose the meteor has an initial speed of $v_0 = 1.0 \times 10^1 \text{ m/s}$. Assume that the meteor started very far away from the earth. Suppose the meteor just grazes the earth. The initial moment arm of the meteor (h on the sketch) is called the impact parameter. You may ignore all other gravitational forces except the earth. The effective target size of the earth as initially seen by the meteor is the area πh^2 .



- a) Draw a force diagram for the forces acting on the meteor.
- b) Can you find a point about which the gravitational torque of the earth's force on the meteor is zero for the entire orbit of the meteor?

Answer:

a),b) The meteor moves under the influence of Earth's gravity. The force is directed along the radius vector from the center of the Earth to the meteor, thus the torque created by this force about the center of the Earth is zero.

- c) What is the initial angular momentum and final angular momentum (when it just grazes the earth) of the meteor?

Answer:

c) The initial angular momentum is $L = mv_0h$ and the final angular momentum is $L = mvr_e$.