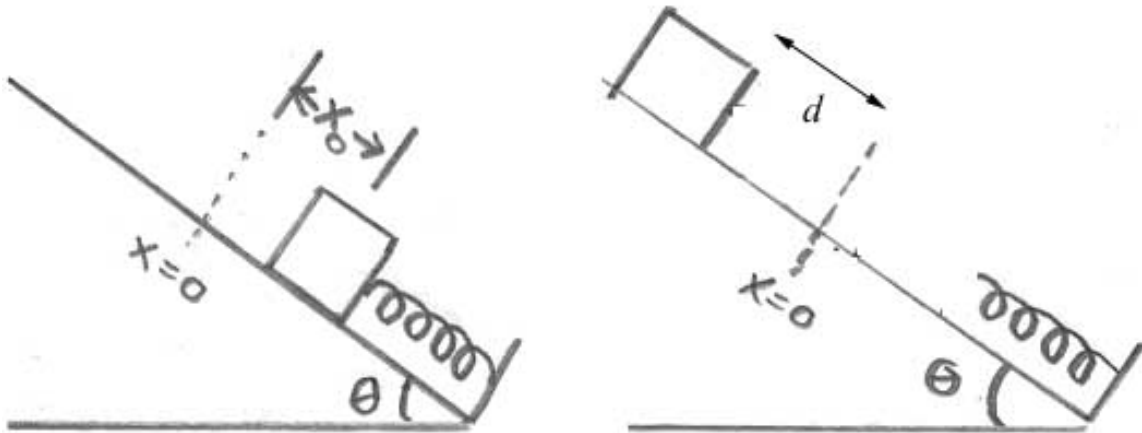


In-Class Problem 21: Spring-Object on an Inclined Plane with Friction

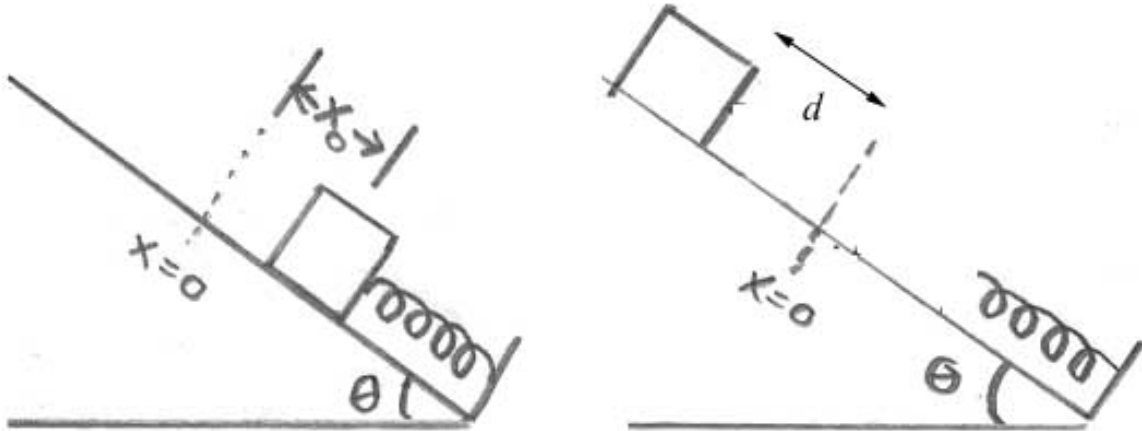
A object of mass m is pushed against a spring at the bottom of a plane that is inclined at an angle θ with respect to the horizontal and held in place with a catch. The spring compresses a distance x_0 and has spring constant k . The catch is released and the object slides up the inclined plane. At $x = 0$ the object detaches from the spring and continues to slide up the inclined plane.



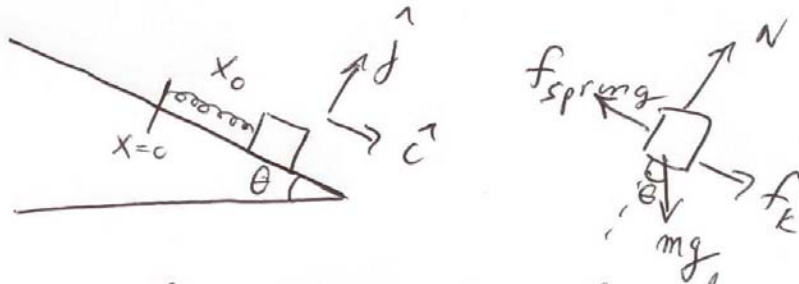
Assume that the incline plane has a coefficient of kinetic friction μ_k . How far up the inclined plane does the object move from the point where the object detaches from the spring?

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Stage 1: The spring is released and the object moves back to equilibrium

$$\Delta K = \int \vec{F}^{total} \cdot d\vec{r}$$

$$\frac{1}{2} m v_1^2 - \frac{1}{2} m v_0^2 = \int_{x_0}^{x=0} -kx dx + \int_{x_0}^{x=0} \mu_k N dx + \int_{x_0}^{x=0} mgsin\theta dx$$

$$\text{since } (\vec{F}^{total})_y = -kx \hat{i} + \mu_k N \hat{i} + mgsin\theta \hat{i}$$

note that $F_y = ma_y = 0 \Rightarrow$
 $N - mg \cos\theta = 0$ or $N = mg \cos\theta$

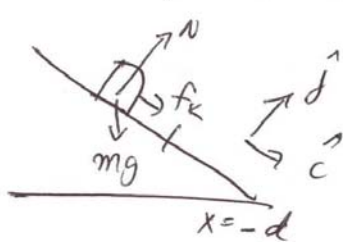
The object is released at rest $v_0 = 0$

$$\text{thus } \frac{1}{2} m v_1^2 = -\int_{x_0}^{x=0} kx dx + \int_{x_0}^{x=0} \mu_k mg \cos\theta dx + \int_{x_0}^{x=0} mgsin\theta dx$$

The three integrals yield

$$\frac{1}{2} m v_1^2 = \frac{1}{2} k x_0^2 - \mu_k m g \cos \theta x_0 - m g \sin \theta x_0 \quad (1)$$

stage 2: object slides up inclined plane until it comes to rest



$$\Delta K = \int_{x=0}^{x=-d} \vec{F} \cdot d\vec{r}$$

$$\Delta K = + \int_0^d \mu_k m g \cos \theta d\theta + \int_0^d m g \sin \theta d\theta$$

$$\frac{1}{2} m v_f^2 - \frac{1}{2} m v_1^2 = (-\mu_k m g \cos \theta - m g \sin \theta) d$$

$$v_f = 0 \quad \text{so}$$

$$\frac{1}{2} m v_1^2 = (\mu_k m g \cos \theta + m g \sin \theta) d \quad (2)$$

Using eq (1) for $\frac{1}{2} m v_1^2$ yields

$$\frac{1}{2} k x_0^2 - (\mu_k m g \cos \theta + m g \sin \theta) x_0 = (\mu_k m g \cos \theta + m g \sin \theta) d$$

solve for $d = \frac{\frac{1}{2} k x_0^2}{(\mu_k m g \cos \theta + m g \sin \theta)} - x_0$