

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 TEAL

Fall Term 2004

Exam 2: Equation Summary

One Dimensional Kinematics:

$$\vec{v} = d\vec{r} / dt, \quad \vec{a} = d\vec{v} / dt$$

$$v_x(t) - v_{x,0} = \int_{t'=0}^{t'=t} a_x(t') dt' \quad x(t) - x_0 = \int_{t'=0}^{t'=t} v_x(t') dt'$$

Constant Acceleration:

$$x(t) = x_0 + v_{x,0}(t - t_0) + \frac{1}{2} a_x (t - t_0)^2 \quad v_x(t) = v_{x,0} + a_x (t - t_0)$$
$$y(t) = y_0 + v_{y,0}(t - t_0) + \frac{1}{2} a_y (t - t_0)^2 \quad v_y(t) = v_{y,0} + a_y (t - t_0)$$

where $x_0, v_{x,0}, y_0, v_{y,0}$ are the initial position and velocities components at $t = t_0$

Newton's Second Law: Force, Mass, Acceleration

$$\vec{F} \equiv m\vec{a} \quad \vec{F}^{total} = \vec{F}_1 + \vec{F}_2 \quad F_x^{total} = ma_x \quad F_y^{total} = ma_y \quad F_z^{total} = ma_z$$

Newton's Third Law:

$$\vec{F}_{1,2} = -\vec{F}_{2,1}$$

Force Laws:

Universal Law of Gravity: $\vec{F}_{1,2} = -G \frac{m_1 m_2}{r_{1,2}^2} \hat{r}_{1,2}$, attractive

Gravity near surface of earth: $\vec{F}_{grav} = m_{grav} \vec{g}$, towards earth

Contact force: $\vec{F}_{contact} = \vec{N} + \vec{f}$, depends on applied forces

Static Friction: $0 \leq f_s \leq f_{s,max} = \mu_s N$ direction depends on applied forces

Kinetic Friction: $f_k = \mu_k N$ opposes motion

Hooke's Law: $F = k|\Delta x|$, restoring

Kinematics Circular Motion: arc length: $s = R\theta$; angular velocity: $\omega = d\theta/dt$
 tangential velocity: $v = R\omega$; angular acceleration: $\alpha = d\omega/dt = d^2\theta/dt^2$; tangential
 acceleration $a_\theta = R\alpha$.

Period: $T = \frac{2\pi R}{v} = \frac{2\pi}{\omega}$; **frequency:** $f = \frac{1}{T} = \frac{\omega}{2\pi}$,

Radial Acceleration: $|a_r| = R\omega^2$; $|a_r| = \frac{v^2}{R}$; $|a_r| = 4\pi^2 R f^2$; $|a_r| = \frac{4\pi^2 R}{T^2}$

Center of Mass: $\vec{R}_{cm} = \sum_{i=1}^{i=N} m_i \vec{r}_i / m^{total} \rightarrow \int_{body} dm \vec{r} / m^{total}$;

Velocity of Center of Mass: $\vec{V}_{cm} = \sum_{i=1}^{i=N} m_i \vec{v}_i / m^{total} \rightarrow \int_{body} dm \vec{v} / m^{total}$

Torque: $\vec{\tau}_S = \vec{r}_{S,P} \times \vec{F}_P$ $|\vec{\tau}_S| = |\vec{r}_{S,P}| |\vec{F}_P| \sin \theta = r_\perp F = r F_\perp$

Static Equilibrium: $\vec{F}_{total} = \vec{F}_1 + \vec{F}_2 + \dots = \vec{0}$; $\vec{\tau}_S^{total} = \vec{\tau}_{S,1} + \vec{\tau}_{S,2} + \dots = \vec{0}$.

Kinetic Energy: $K = \frac{1}{2}mv^2$; $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$

Work: $W = \int_{r_0}^{r_f} \vec{F} \cdot d\vec{r}$; **Work- Kinetic Energy:** $W^{total} = \Delta K$

Power: $P = \vec{F} \cdot \vec{v} = dK/dt$

Potential Energy: $\Delta U = -W_{conservative} = -\int_A^B \vec{F}_c \cdot d\vec{r}$

Potential Energy Functions with Zero Points:

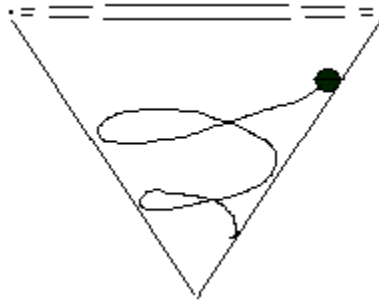
Constant Gravity: $U(y) = mgy$ with $U(y_0 = 0) = 0$.

Inverse Square Gravity: $U_{gravity}(r) = -\frac{Gm_1m_2}{r}$ with $U_{gravity}(r_0 = \infty) = 0$.

Hooke's Law: $U_{spring}(x) = \frac{1}{2}kx^2$ with $U_{spring}(x = 0) = 0$.

Work- Mechanical Energy: $W_{nc} = \Delta K + \Delta U^{total} = \Delta E_{mech} = (K_f + U_f^{total}) - (K_0 + U_0^{total})$,

- c) **(4 points)** A ball is rolling without slipping in a spiral path down the inside of a hollow cone.



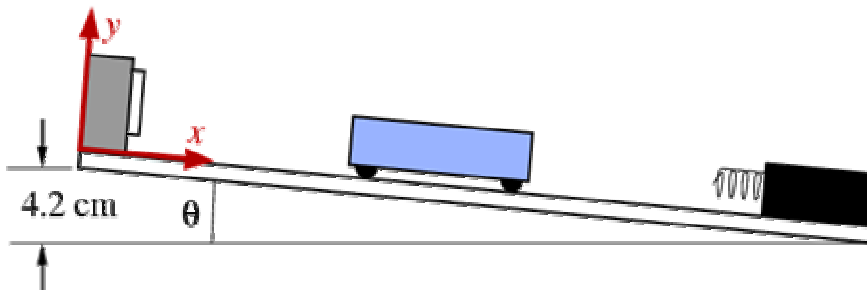
The work done by the inner surface of the cone on the ball is

1. positive
2. zero
3. negative
4. Impossible to determine

~~d) **(4 points)** Suppose you want to ride your mountain bike up a steep hill. Two paths lead from the base to the top, one twice as long as the other. You start and end at rest. Compared to the average force you would exert if you took the short path, the average force you exert along the longer path is~~

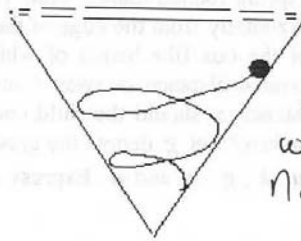
- ~~1. one quarter as much.~~
- ~~2. one third as much.~~
- ~~3. one half as much.~~
- ~~4. the same.~~
- ~~5. undetermined it depends on the time taken.~~

~~e) **(4 points)** When the cart maximally compresses the spring at the bottom of the track, the cart's~~



- ~~1. velocity and acceleration are zero.~~
- ~~2. velocity is nonzero but its acceleration is zero.~~
- ~~3. acceleration is nonzero, but its velocity is zero.~~
- ~~4. velocity and acceleration are both nonzero.~~

- c) (4 points) A ball is rolling without slipping in a spiral path down the inside of a hollow cone.



Since the ball rolls without slipping contact point is at rest, friction does no work.

Normal force \perp displacement so normal force does no work.

The work done by the inner surface of the cone on the ball is

1. positive
2. zero
3. negative
4. impossible to determine

- d) (4 points) Suppose you want to ride your mountain bike up a steep hill. Two paths lead from the base to the top, one twice as long as the other. You start and end at rest. Compared to the average force you would exert if you took the short path, the average force you exert along the longer path is

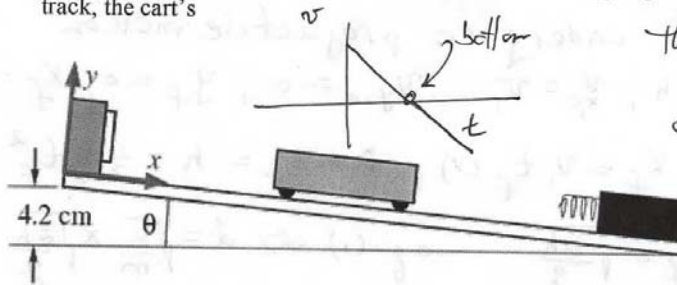
$$W_{\text{ext}} = mgh = (F_{\text{ave}})_1 x_1 = (F_{\text{ave}})_2 x_2$$

$$\text{Since } x_2 = 2x_1 \Rightarrow$$

$$(F_{\text{ave}})_2 = (F_{\text{ave}})_1 \frac{x_1}{x_2} = \frac{(F_{\text{ave}})_1}{2}$$

1. one quarter as much.
2. one third as much.
3. one half as much.
4. the same.
5. undetermined-it depends on the time taken.

- e) (4 points) When the cart maximally compresses the spring at the bottom of the track, the cart's



The spring force is maximum.

The cart comes to rest but

the spring force is accelerating the cart in the negative x-direction

1. velocity and acceleration are zero.
2. velocity is nonzero but its acceleration is zero.
3. acceleration is nonzero, but its velocity is zero.
4. velocity and acceleration are both nonzero.