

Problem 8: (Archimedes's Principle)

A cylindrical beaker of mass $m_b = 1.3 \text{ kg}$ contains $1.5 \times 10^3 \text{ ml}$ of water. The beaker is placed on a scale and then a rock of mass $m_r = 2.2 \text{ kg}$, suspended by a massless string, is totally immersed in the water. The water level rises by 1.5 cm . The diameter of the beaker is 0.2 m .

- a) What mass does the scale measure before the rock is lowered into the water?
- b) What mass does the scale measure after the rock is lowered into the water?
- c) What is the density of the rock?

A plastic hollow sphere of the same volume as the rock has density $\rho_s = 0.6 \times 10^3 \text{ kg/m}^3$. The sphere is held just below the surface of the water by a rope that is attached to the bottom of the beaker.

- d) What mass does the scale now read?

Problem 9: (Planetary Orbits)

A satellite of mass m_1 is in an elliptical orbit around a planet of mass m_2 whose center is at F, one of the two foci of the ellipse ($m_2 \gg m_1$). P is the point of closest approach of the two masses and is a distance r_p from the foci F. The satellite's speed at the point P is v_p . The two masses are furthest apart at the point A which is a distance $r_A = 5 r_p$.

- a) What is the speed of the satellite at the point A?
- b) What is the energy of the satellite?

Problem 10: (Energy, Force, and Kinematics)

A child's playground slide is 5.0 m in length and is at an angle of 2.0×10^1 degrees with respect to the ground. A child of mass $2.0 \times 10^1 \text{ kg}$ starts from rest at the top of the slide. The coefficient of sliding friction for the slide is $\mu_k = 0.2$.

- a) What is the total work done by the friction force on the child?
- b) What is the speed of the child at the bottom of the slide?
- c) How long does the child take to slide down the ramp?

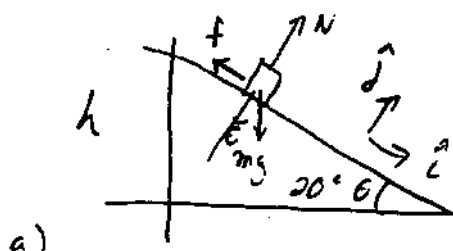
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$$N - mg \cos \theta = 0 \quad (1)$$

$$mg \sin \theta - f = ma \quad (2)$$

$$f = \mu_k N = \mu_k mg \cos \theta \quad (3)$$

$$\Rightarrow W_{\text{friction}} = -f_k d = -\mu_k mg \cos \theta d$$

$$= -(0.2)(20 \text{ kg})(10 \frac{\text{m}}{\text{s}^2})(\cos 20^\circ)(5.0 \text{ m}) = -1.9 \times 10^2 \text{ J}$$

b)

$$\Delta K + \Delta U = W_{\text{friction}}$$

$$\left(\frac{1}{2}mv^2 - 0\right) + (0 - mgd \sin \theta) = -f_k d$$

$$\Rightarrow v = \left(\frac{2}{m}(mgd \sin \theta - f_k d)\right)^{1/2}$$

$$= \left(\frac{2}{m}(mgd \sin \theta - \mu_k mg \cos \theta d)\right)^{1/2} = (2gd(\sin \theta - \mu_k \cos \theta))^{1/2}$$

$$= \left((2)(10 \frac{\text{m}}{\text{s}^2})(5.0 \text{ m})(\sin 20^\circ - 0.2 \cos 20^\circ)\right)^{1/2} = 3.9 \text{ m/s}$$

c) From eq (2) $a = g \sin \theta - \mu_k g \cos \theta = g(\sin \theta - \mu_k \cos \theta)$

$$d = \frac{1}{2}at^2 \Rightarrow t = \left(\frac{2d}{a}\right)^{1/2} = \left(\frac{2d}{g(\sin \theta - \mu_k \cos \theta)}\right)^{1/2} = 2.5 \text{ sec}$$

Problem 11: Experiment flow

Blood flow through the vascular system of the human body is controlled by several factors. The flow is directly proportional to the pressure differential ΔP , between any two points in the system.

$$\Delta P \propto I.$$

Let L be the length of the vessel, d the diameter of the vessel, and η the viscosity. Let $v(r)$ denote the velocity of the fluid at a distance r from the central axis. Then the laminar flow of a blood in a cylindrical vessel can be modelled by

$$v(r) = \frac{(\Delta P)(\frac{d^2}{4} - r^2)}{4\eta L}$$

where r is the distance from the center of the vessel.

There is a change of velocity with respect to distance from the center across the vessel, $\frac{dv}{dr}$. A viscous force $F_{\text{viscous}} = F$, acts on any cylindrical element of blood in the direction opposite the flow due to the slower moving blood outside the element. The magnitude of the viscous force is given by

$$F = -\eta A \frac{dv}{dr} = -\eta 2\pi r L \frac{dv}{dr},$$

where $A = 2\pi r L$ is the area of the cylindrical element.

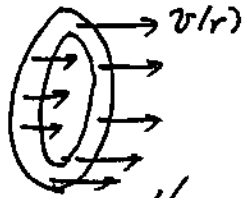
Changes in the content of the blood can vary the viscosity. Also the arteries tend to more closely approximate the ideal model of a cylindrical pipe. Veins are more apt to change their geometry.

- Calculate the average velocity, v_{ave} , through a vessel. What is the ratio between the average velocity and the velocity at the center $r = 0$.
- Calculate the total flow I through the vessel.
- Calculate the force on the walls of the vessel (at $r = d/2$).
- Using your result from part c), what is the net force on the vessel?

$$a) \quad v_{\text{ave}} = \frac{1}{\pi(\frac{d}{2})^2} \int_{r=0}^{r=d/2} v(r) 2\pi r dr$$

where $2\pi r dr$ is the area of a ring of

radius r and width dr .



$$v_{ave} = \frac{1}{\text{area}} \int_0^{d/2} v(r) 2\pi r dr$$

$$v_{ave} = \frac{1}{\pi(\frac{d}{2})^2} \int_0^{d/2} \frac{\Delta P ((\frac{d}{2})^2 - r^2)}{4\eta L} 2\pi r dr$$

$$= \frac{\Delta P 2\pi}{\pi(\frac{d}{2})^2 4\eta L} \int_0^{d/2} ((\frac{d}{2})^2 - r^2) r dr$$

$$= \frac{\Delta P 2\pi}{\pi(\frac{d}{2})^2 4\eta L} \left((\frac{d}{2})^2 \frac{r^2}{2} \Big|_0^{d/2} - \frac{r^4}{4} \Big|_0^{d/2} \right)$$

$$= \frac{\Delta P}{(\frac{d}{2})^2 2\eta L} \left(\frac{1}{2} (\frac{d}{2})^4 - \frac{1}{4} (\frac{d}{2})^4 \right) = \frac{\Delta P}{(\frac{d}{2})^2 2\eta L} \frac{1}{4} (\frac{d}{2})^4$$

$$v_{ave} = \frac{\Delta P}{8\eta L} \left(\frac{d}{2}\right)^2$$

$$\frac{v_{ave}}{v(r=0)} = \frac{\frac{\Delta P}{8\eta L} \left(\frac{d}{2}\right)^2}{\frac{\Delta P}{4\eta L} \left(\frac{d}{2}\right)^2} = \frac{1}{2}$$

$$(b) I = \int_0^{d/2} v(r) 2\pi r dr = (\text{area}) v_{ave}$$

$$I = \frac{\Delta P}{8\eta L} \left(\frac{d}{2}\right)^2 \pi \left(\frac{d}{2}\right)^2 \Rightarrow I \sim \Delta P$$

c) The force on the wall of the vessel has magnitude

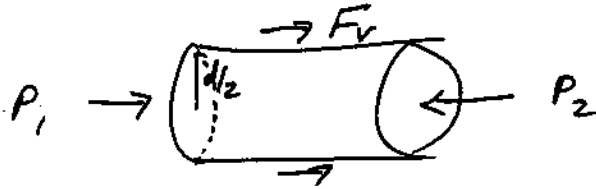
$$F_v = \left| \eta A \frac{dv}{dr} \right| = \left| \eta 2\pi \frac{d}{2} L \frac{dv}{dr} \right|_{r=d/2}$$

$$\left. \frac{dv}{dr} \right|_{r=R} = \frac{\Delta P}{4\eta L} (-2r) \Big|_{r=d/2} = -\frac{\Delta P}{4\eta L} 2 \frac{d}{2}$$

$$F_v = \left| -\eta 2\pi \frac{d}{2} L \frac{\Delta P}{4\eta L} 2 \frac{d}{2} \right|$$

$$F_v = \Delta P \pi \left(\frac{d}{2} \right)^2$$

d) The net force on the vessel is due to the viscous force and the pressure differential at the ends



$$\pi \left(\frac{d}{2} \right)^2 (P_1 - P_2) + F_v = 0$$

$$\pi \left(\frac{d}{2} \right)^2 (-\Delta P) + F_v = 0$$

$$\pi \left(\frac{d}{2} \right)^2 \Delta P = F_v$$

agreeing with the result

in part c).