

Problem 2 (32 points).

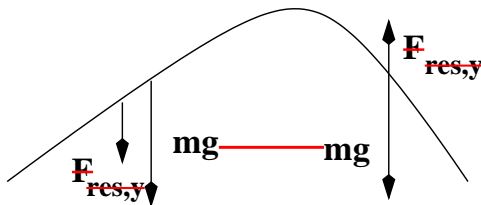
- a. (6) I throw an object of mass m up from the ground at an angle of 45° with the vertical. There is a substantial air drag on the object. It reaches its highest point after 2 sec. Will it take longer or shorter than 2 sec to fall back to the ground or will it take the same amount of time? Explain your answer clearly.
[$g = 10 \text{ m/sec}^2$]
- b. (6) A pendulum is hanging from the ceiling of an elevator. Its period (at small angles) is T sec when the elevator is at rest. We now accelerate the elevator downwards with 5 m/sec^2 . What is the period now? Be quantitative.
[$g = 10 \text{ m/sec}^2$]
- c. (6) We release at zero speed an oil drop of radius r in air at 1 atmosphere. The density of the oil is ρ . How small should the oil drop be so that the drag force is dominated by the viscous term which is proportional with the speed? C_1 and C_2 are the coefficients (for 1 atmosphere air) for the viscous and the pressure term, respectively.

A particle moves in one dimension as a function of time: $x = -0.3 \sin(2t + \pi/4)$.
 x is in meters, t in sec.

- d. (6) What is the frequency (in Hz) of this simple harmonic oscillation?
- e. (8) What are the times (in sec) at which the speed of the particle is maximum?

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- 6 pts **a)** ~~As the object goes up, the component of the resistive force in the vertical (y) direction is in the same direction as the gravitational force mg (see figure).~~



~~When the object is on the way down, the y component of the resistive force is in the opposite direction. Thus, the component of the force in the y direction is smaller on the way down than on the way up. Thus, it will take **longer than 2 sec** to come down.~~

- 6 pts **b)** For zero acceleration, $T = 2\pi\sqrt{\frac{l}{g}}$. For an acceleration a downwards, $T' = 2\pi\sqrt{\frac{l}{g-a}}$. If a were 10 m/sec², we'd have free fall, and the pendulum would be weightless (no oscillations, $T \rightarrow \infty$). Since $a = 5$ m/sec², $T' = T\sqrt{2}$.

- 6 pts **c)** ~~The viscous term will dominate when $v_{\text{term}} \ll v_{\text{crit}}$ where $v_{\text{crit}} = \frac{C_1}{C_2 r}$. When the viscous term dominates, $v_{\text{term}} = \frac{mg}{C_1 r}$. The mass of the oil drop is given by $m = \frac{4}{3}\pi r^3 \rho$. Thus,~~

$$\Rightarrow r \ll \left(\frac{3C_1^2}{4\pi\rho g C_2} \right)^{\frac{1}{3}}$$

- 6 pts **d)** $\omega = 2$, $T = \frac{2\pi}{\omega}$, $f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{\pi}$ Hz

- 8 pts **e)** ~~The speed is maximum when $x = 0$, i.e., when $\sin(2t + \frac{\pi}{4}) = 0$~~

~~This happens when $2t + \frac{\pi}{4} = n\pi$ where $n = 0, \pm 1, \pm 2, \pm 3, \dots$~~

$$\Rightarrow t = \frac{\pi}{2}(n - \frac{1}{4})$$

~~Alternatively, $v = -0.6 \cos(2t + \frac{\pi}{4})$, and the max speed is when $\cos(2t + \frac{\pi}{4}) = \pm 1$. This leads to the same times.~~