

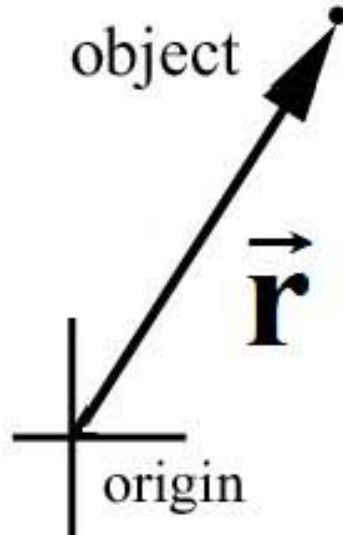
Applying Newton's Second Law

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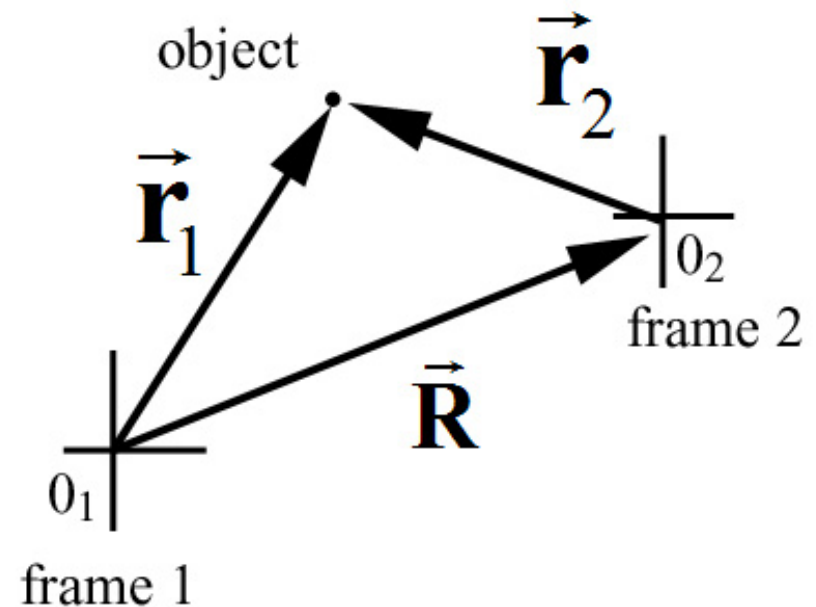
Reference Frame

Coordinate system with an observer placed at origin is a '**reference frame**' in which the position, velocity, and acceleration of objects are mathematically defined



Relative Reference Frames

- Two reference frames
- Origins need not coincide
- Moving object
- Position vectors in different frames



$$\vec{r}_1 = \vec{R} + \vec{r}_2$$

Relatively Inertial Reference Frames

Relative velocity between the two reference frames is constant

$$\vec{V} = d\vec{R}/dt$$

The relative acceleration is zero

$$\vec{A} = d\vec{V}/dt = \vec{0}$$

Frames are called *relatively inertial reference frames*

Law of Addition of Velocities

Suppose object is moving, then observers in different reference frames will measure different velocities

- velocity of the object in Frame 1: $\vec{\mathbf{v}}_1 = d\vec{\mathbf{r}}_1/dt$
- velocity of the object in Frame 2: $\vec{\mathbf{v}}_2 = d\vec{\mathbf{r}}_2/dt$
- velocity of an object in two different reference frames

$$\vec{\mathbf{v}}_1 = \vec{\mathbf{V}} + \vec{\mathbf{v}}_2$$

Acceleration in Relatively Inertial Reference Frames

Suppose object is accelerating, then observers in different relatively reference frames will measure the same acceleration

- acceleration of the object in Frame 1: $\vec{\mathbf{a}}_1 = d\vec{\mathbf{v}}_1/dt$
- acceleration of the object in Frame 2: $\vec{\mathbf{a}}_2 = d\vec{\mathbf{v}}_2/dt$
- Relative acceleration of frames is zero $\vec{\mathbf{A}} = d\vec{\mathbf{V}}/dt = \vec{\mathbf{0}}$
- acceleration of an object in two different reference frames $\vec{\mathbf{a}}_1 = \vec{\mathbf{A}} + \vec{\mathbf{a}}_2 = \vec{\mathbf{a}}_2$

Newton's First Law: Law of Inertia

Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it

Application: Newton's Second Law in either frame is

$$\vec{\mathbf{F}}_1^T = m\vec{\mathbf{a}}_1 \qquad \vec{\mathbf{F}}_2^T = m\vec{\mathbf{a}}_2$$

Relatively inertial reference frames have same accelerations.

$$\vec{\mathbf{a}}_1 = \vec{\mathbf{a}}_2$$

So use the same forces

$$\vec{\mathbf{F}}_1^T = \vec{\mathbf{F}}_2^T$$

So it does not matter which frame you choose to describe the problem.

Principle of Relativity

For observers moving in two different reference frames, no mechanical force can distinguish which observer is at rest and which observer is moving

Einstein extended this to all physical phenomena.

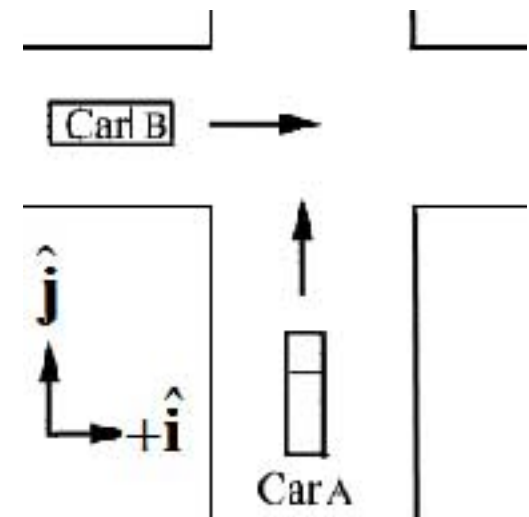
The laws of physics are the same in all relatively inertial reference frames,

In particular, Einstein extended the Galilean principle of relativity to electromagnetism and optics which describe the theory of light

In-Class Problem 7: Law of Addition of Velocities

Suppose two cars, Car A, and Car B, are traveled along roads that are perpendicular to each other. An observer is at rest with respect to the ground. A second observer is in Car A. According to the observer on the ground, Car A is moving with a velocity $\vec{v}_A = v_A \hat{j}$, and Car B is moving with a magnitude of velocity $\vec{v}_B = v_B \hat{i}$.

What is the velocity of Car B according to the observer in Car A? Express your answer both as components of the velocity vector, and direction and magnitude of the velocity vector.

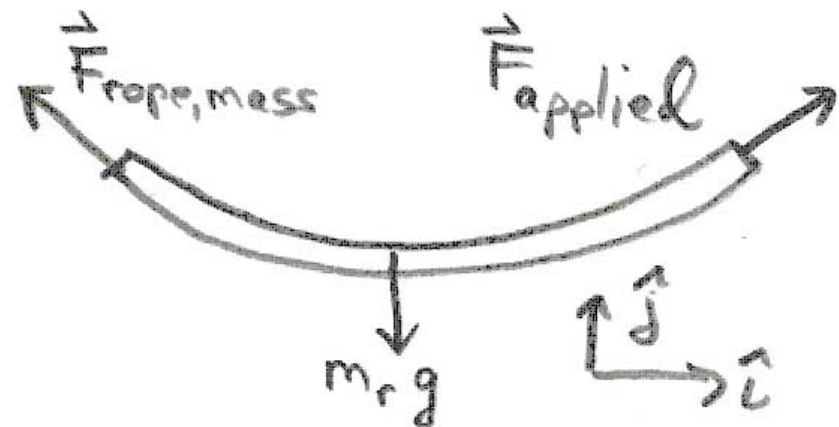
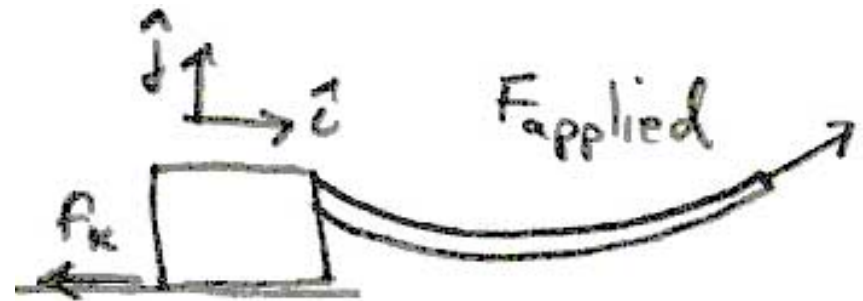


Tension

Consider a rope pulling a mass.

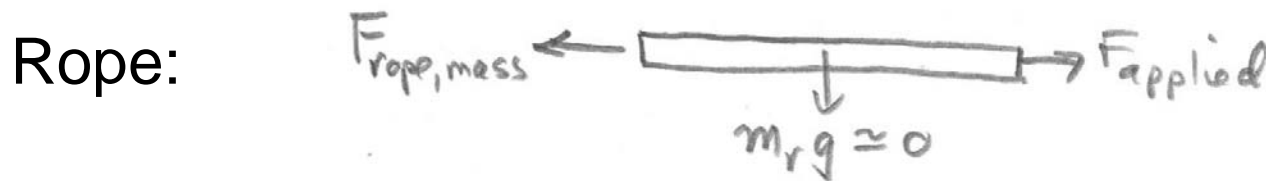
How do we define 'tension' in a rope?

Force Diagram on Rope

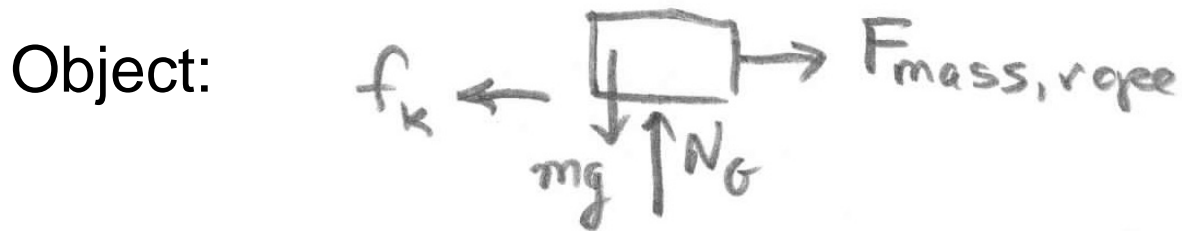


Tension: Massless Rope

- **Assumption:** Assume that the mass of the rope is small so that we can ignore all y-components of the applied force and the force of the object on the rope.



$$F_{applied} - F_{rope, mass} = m_r a_{r, x}$$



$$F_x^{total} = F_{mass, rope} - f_{kinetic}$$

$$F_y^{total} = N - mg = 0$$

Equations of Motion

Friction Force Law: $f_{kinetic} = \mu_k N$

Newton's Third Law: $\vec{\mathbf{F}}_{mass,rope} = -\vec{\mathbf{F}}_{rope,mass}$

$$F_{rope,mass} = F_{mass,rope}$$

Newton's Second Law: $F_{mass,rope} - \mu_k mg = ma_{m,x}$

$$F_{applied} - F_{mass,rope} = m_r a_{r,x}$$

Inextensible Rope Assumption: $a_{m,x} = a_{r,x} \equiv a$

Special Cases

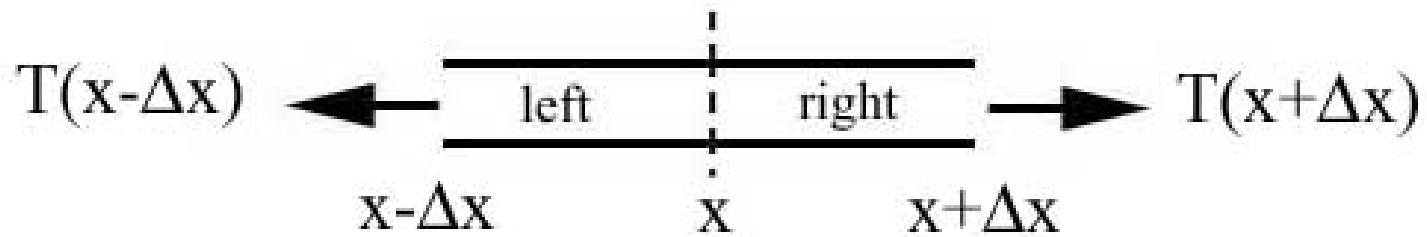
1. Pulling the object at a constant velocity: $a = 0$

2. Rope is essentially massless: $m_r \cong 0$

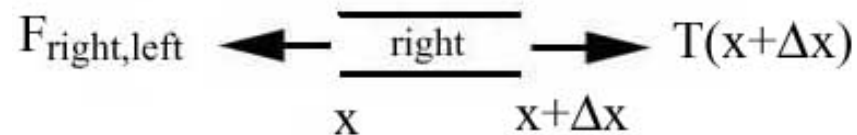
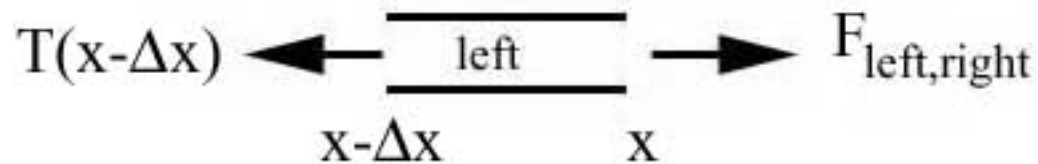
$$F_{applied} - F_{mass,rope} = 0$$

Pulling force is transmitted through the rope

Tension in a Rope



The *tension* in a rope at a distance x from one end of the rope is the magnitude of the action -reaction pair of forces acting at that point ,



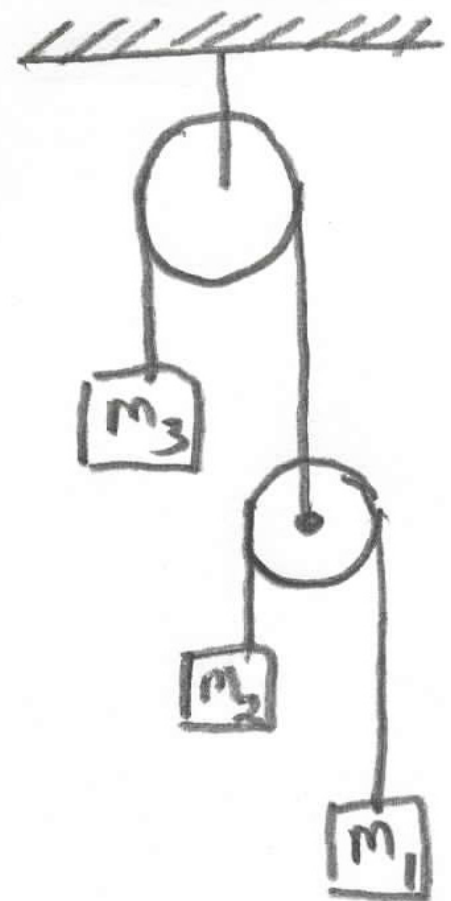
$$T(x) = \left| \vec{\mathbf{F}}_{\text{left, right}}(x) \right| = \left| \vec{\mathbf{F}}_{\text{right, left}}(x) \right|$$

Constraint Condition in Pulley Systems

Consider the arrangement of pulleys and objects shown in the figure. Draw force diagrams on each object.

How are the accelerations of the objects related?

Solve for the accelerations of the objects and the tensions in the ropes.



Force Law: Newtonian Induction

- Definition of force has no predictive content
- Need to measure the acceleration and the mass in order to define the force
- **Force Law:** Discover experimental relation between force exerted on object and change in properties of object
- **Induction:** Extend force law from finite measurements to all cases within some range creating a **model**
- **Second Law can now be used to predict motion!**
- If prediction disagrees with measurement adjust model.