

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 TEAL

Fall Term 2004

Exam 1: Equation Summary

One Dimensional Kinematics:

$$\vec{v} = d\vec{r} / dt, \quad \vec{a} = d\vec{v} / dt \qquad v_x(t) - v_{x,0} = \int_{t'=0}^{t'=t} a_x(t') dt' \qquad x(t) - x_0 = \int_{t'=0}^{t'=t} v_x(t') dt'$$

Constant Acceleration:

$$x(t) = x_0 + v_{x,0}(t - t_0) + \frac{1}{2} a_x (t - t_0)^2 \qquad v_x(t) = v_{x,0} + a_x (t - t_0)$$
$$y(t) = y_0 + v_{y,0}(t - t_0) + \frac{1}{2} a_y (t - t_0)^2 \qquad v_y(t) = v_{y,0} + a_y (t - t_0)$$

where $x_0, v_{x,0}, y_0, v_{y,0}$ are the initial position and velocities components at $t = t_0$

Law of Addition of Velocities : $\vec{v}_{object, frame 1} = \vec{v}_{object, frame 2} + \vec{V}_{relative}$

Newton's Second Law: Force, Mass, Acceleration

$$\vec{F} \equiv m\vec{a} \quad \vec{F}^{total} = \vec{F}_1 + \vec{F}_2 \quad F_x^{total} = ma_x \quad F_y^{total} = ma_y \quad F_z^{total} = ma_z$$

Newton's Third Law: $\vec{F}_{1,2} = -\vec{F}_{2,1}$

Force Laws:

Universal Law of Gravity: $\vec{F}_{1,2} = -G \frac{m_1 m_2}{r_{1,2}^2} \hat{r}_{1,2}$, attractive

Gravity near surface of earth: $\vec{F}_{grav} = m_{grav} \vec{g}$, towards earth

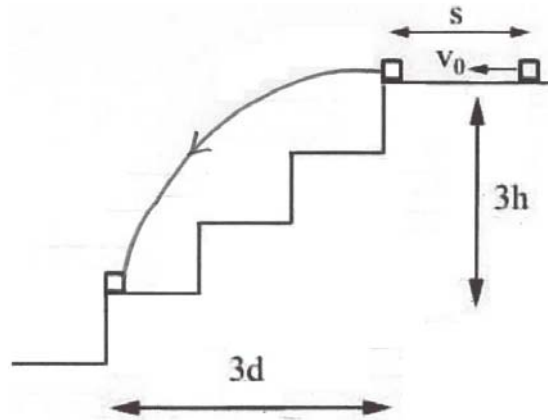
Contact force: $\vec{F}_{contact} = \vec{N} + \vec{f}$, depends on applied forces

Static Friction: $0 \leq f_s \leq f_{s,max} = \mu_s N$ direction depends on applied forces

Kinetic Friction: $f_k = \mu_k N$ opposes motion

Hooke's Law: $F = k |\Delta x|$, restoring

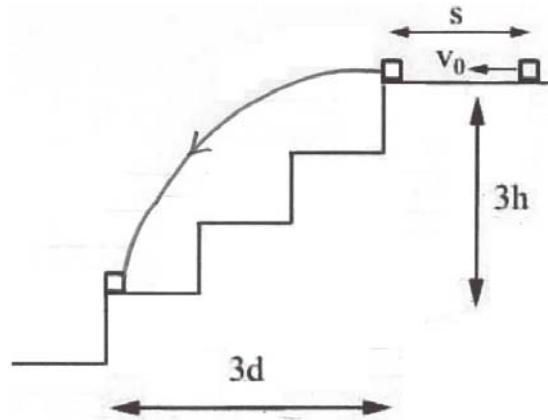
Problem 3: (30 points) An object of given mass m starts with a given velocity v_0 and slides an unknown distance s along a floor and then off the top of a staircase. The goal of this problem is to find the distance s . The coefficient of kinetic friction between the object and the floor is given by μ_k . The object strikes at the far end of the third stair. Each stair has a given rise of h and a given run of d . You may neglect air resistance and assume the gravitational constant is given as g .



- Briefly describe how you intend to model the motion of the object? What are the given quantities in this problem?
- What is the velocity of the object just as it leaves the top of the staircase? Express your answer in terms of the given quantities only.
- What is the acceleration of the object while it slides on the floor? Express your answer in terms of the given quantities only.
- What is the unknown distance s that the object slides along the floor? Express your answer in terms of the given quantities only.

Answer:

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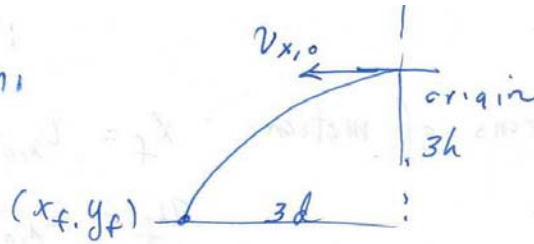


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Answer:

model : two stages
 stage (1) : horizontal motion with deceleration due to friction
 stage (2) : projectile motion: $a_{hor} = 0$, $a_{ver} = -g$
 with pos. vertical direction upward
 givens: m, v_0, μ_k, h, d, g

projectile motion:



initial conditions: $x_0 = 0, y_0 = 0, v_{y,0} = 0$
 $v_{x,0}$ unknown

final conditions: $x_f = 3d, y_f = -3h$

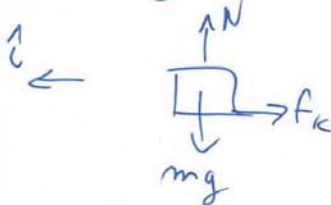
equations of motion: $x_f = v_{x,0} t_f$
 $y_f = -\frac{1}{2} g t_f^2$

become (1) $3d = v_{x,0} t_f$

(2) $-3h = -\frac{1}{2} g t_f^2 \Rightarrow t_f = \sqrt{\frac{6h}{g}}$

eq (1) $\Rightarrow v_{x,0} = \frac{3d}{t_f} = \frac{3d}{\sqrt{6h/g}} = d \sqrt{\frac{3g}{2h}}$

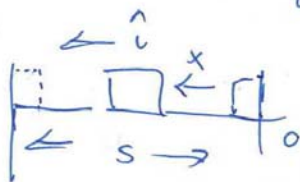
horizontal motion:



	\vec{F}	$= m \vec{a}$
\hat{i}	$-f_c$	ma
\hat{j}	$N - mg$	0

$$f_k = \mu_k N = \mu_k mg \Rightarrow -\mu_k mg = ma \Rightarrow$$

$$a = -\mu_k g$$



initial conditions: $x_0 = 0, v_{x,0} = v_0$

final conditions: $x_f = s, v_f = d \sqrt{\frac{3g}{2h}}$

equations of motion: $x_f = v_{x,0} t_f + \frac{1}{2} a t_f^2$ (3)

$$v_f = v_{x,0} + a t_f \quad (4)$$

$$\text{eq (4)} \Rightarrow t_f = \frac{v_f - v_{x,0}}{a}$$

$$\begin{aligned} \text{eq (3)} \Rightarrow x_f &= v_{x,0} \left(\frac{v_f - v_{x,0}}{a} \right) + \frac{1}{2} a \left(\frac{v_f - v_{x,0}}{a} \right)^2 \\ &= \frac{v_{x,0} v_f}{a} - \frac{v_{x,0}^2}{a} + \frac{1}{2a} (v_f^2 + v_{x,0}^2 - 2v_f v_{x,0}) \end{aligned}$$

$$x_f = \frac{1}{2a} (v_f^2 - v_{x,0}^2)$$

$$S = \frac{1}{(2)(-14 \text{ kg})} \left(d^2 \frac{3}{2} \frac{g}{h} - v_0^2 \right)$$