

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 TEAL

Fall Term 2004

Exam 1: Equation Summary

One Dimensional Kinematics:

$$\vec{v} = d\vec{r} / dt, \quad \vec{a} = d\vec{v} / dt$$

$$v_x(t) - v_{x,0} = \int_{t'=0}^{t'=t} a_x(t') dt' \quad x(t) - x_0 = \int_{t'=0}^{t'=t} v_x(t') dt'$$

Constant Acceleration:

$$x(t) = x_0 + v_{x,0}(t - t_0) + \frac{1}{2} a_x (t - t_0)^2 \quad v_x(t) = v_{x,0} + a_x (t - t_0)$$
$$y(t) = y_0 + v_{y,0}(t - t_0) + \frac{1}{2} a_y (t - t_0)^2 \quad v_y(t) = v_{y,0} + a_y (t - t_0)$$

where $x_0, v_{x,0}, y_0, v_{y,0}$ are the initial position and velocities components at $t = t_0$

Newton's Second Law: Force, Mass, Acceleration

$$\vec{F} \equiv m\vec{a} \quad \vec{F}^{total} = \vec{F}_1 + \vec{F}_2 \quad F_x^{total} = ma_x \quad F_y^{total} = ma_y \quad F_z^{total} = ma_z$$

Newton's Third Law:

$$\vec{F}_{1,2} = -\vec{F}_{2,1}$$

Force Laws:

Universal Law of Gravity: $\vec{F}_{1,2} = -G \frac{m_1 m_2}{r_{1,2}^2} \hat{r}_{1,2}$, attractive

Gravity near surface of earth: $\vec{F}_{grav} = m_{grav} \vec{g}$, towards earth

Contact force: $\vec{F}_{contact} = \vec{N} + \vec{f}$, depends on applied forces

Static Friction: $0 \leq f_s \leq f_{s,max} = \mu_s N$ direction depends on applied forces

Kinetic Friction: $f_k = \mu_k N$ opposes motion

Hooke's Law: $F = k|\Delta x|$, restoring

6. A basketball player is jumping vertically upward in order to land a shot. Her legs are flexed and pushing on the floor so that her body is accelerated upward.

- a) Draw free-body diagrams of the player's body and Earth. Show the relative magnitudes of the various forces and describe each in words (i.e., contact, gravitational, etc., and indicate which object exerts that force and on what). Identify the action-reaction pairs.
- b) Repeat this exercise for the situation immediately after the player's body breaks contact with the floor.
- c) Finally, consider, in the same manner, the situation at the top of the jump.

Analytic Questions: Show all your work.

Problem 1 *One dimensional kinematics: track event*

During a track event two runners, Bob, and Jim, round the last turn and head into the final stretch with Bob a distance d in front of Jim. They are both running with the same velocity v_0 . When the finish line is a distance s away from Jim, Jim accelerates at a constant a_j until he catches up to Bob and passes him. Jim then continues at a constant speed until he reaches the finish line.

- a) How long did it take Jim to catch Bob?
- b) How far did Jim still have to run when he just caught up to Bob?
- c) How long did Jim take to reach the finish line after he just caught up to Bob?

Bob starts to accelerate at a constant a_B at the exact moment that Jim catches up to him, and accelerates all the way to the finish line and crosses the line exactly when Jim does. Assume Bob's acceleration is constant.

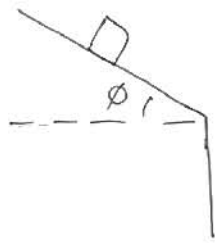
- d) What is Bob's acceleration?
- e) What is Bob's velocity at the finish line? Who is running faster?

Problem 2: *Second Law and projectile motion*

An object with mass m slides down a roof a distance d that is inclined at an angle of ϕ to the horizontal. The contact surface between the roof and object has a coefficient of kinetic friction μ_k . The edge of the bottom of the roof is at a height h above the ground. What is the horizontal displacement from the edge of the roof to the point where the object hits the ground?

Problem 2: *Second Law and projectile motion*

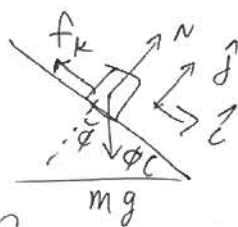
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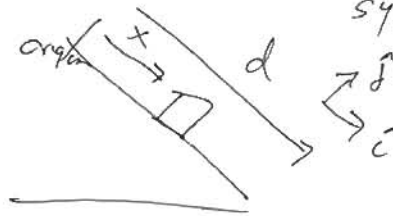
two motions

- 1) sliding down roof
- 2) free fall in air

1) sliding down roof: choose coordinate system



Free body diagram



$$\vec{F} = m\vec{a}$$

$$\hat{i}: mg \sin \phi - f_k = m a$$

$$\hat{j}: N - mg \cos \phi = 0 \Rightarrow N = mg \cos \phi$$

$$\text{kinetic friction: } f_k = \mu_k N = \mu_k mg \cos \phi$$

$$\Rightarrow mg \sin \phi - \mu_k mg \cos \phi = m a$$

$$a = g(\sin \phi - \mu_k \cos \phi)$$

$$x = x_0 + v_{x,0} t + \frac{1}{2} a t^2, \quad v = a t$$

$$x_0 = 0 \text{ choice}, \quad v_{x,0} = 0 \text{ starts from rest}$$

$x = d$, edge of roof

$$d = \frac{1}{2} a t^2 \Rightarrow t = \sqrt{\frac{2d}{a}}$$

$$v = a t = a \sqrt{\frac{2d}{a}} = \sqrt{2da}$$

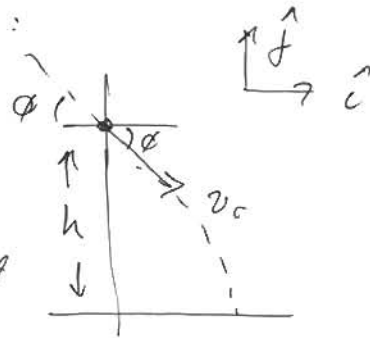
$$= \sqrt{2d(g \sin \phi - \mu_k \cos \phi)}$$

stage 2) free fall:

initial conditions

coordinate system:

origin at point object
leaves roof



initial conditions: $x_0 = y_0 = 0$

$$v_{x,0} = v_0 \cos \phi, \quad v_{y,0} = -v_0 \sin \phi$$

$$\text{where } v_0 = \sqrt{2d(g \sin \phi - \mu_k \cos \phi)}$$

$$y_f = v_{y,0} t_f - \frac{1}{2} g t_f^2$$

$$x_f = v_{x,0} t_f$$

$$v_{y,0} = -v_0 \sin \phi, \quad y_f = -h$$

$$\frac{1}{2} g t_f^2 - v_{y,0} t_f + y_f = 0$$

$$t_f^2 + \frac{2}{g} v_0 \sin \phi t_f - \frac{h}{g} = 0$$

$$t_f = \frac{1}{2} \left(-\frac{2}{g} v_0 \sin \phi + \left(\frac{4v_0^2 \sin^2 \phi}{g^2} + \frac{8h}{g} \right)^{1/2} \right)$$

$t_f > 0 \Rightarrow$ positive root

$$t_f = \frac{1}{2} \left(-\frac{2}{g} v_0 \sin \phi + \left(\frac{4v_0^2 \sin^2 \phi}{g^2} + \frac{8h}{g} \right)^{1/2} \right)$$

$$x_f = v_0 \cos \phi t_f$$

$$x_f = \frac{v_0 \cos \phi}{2} \left(-\frac{2}{g} v_0 \sin \phi + \left(\frac{4v_0^2 \sin^2 \phi}{g^2} + \frac{8h}{g} \right)^{1/2} \right)$$