

**Problem 2 (34 points)**

A particle is moving in three dimensions. Its position vector is given by:

$$\mathbf{r} = 6\hat{x} + (3 + 4t)\hat{y} - (3 + 2t - t^2)\hat{z}.$$

Distances are in meters, and the time,  $t$ , in seconds.

- a) (6) What is the velocity vector at  $t = +3$ ?
  - b) (6) What is the speed (in m/sec) at  $t = +3$ ?
  - c) (6) What is the acceleration vector and what is its magnitude (in m/sec<sup>2</sup>) at  $t = +3$ ?
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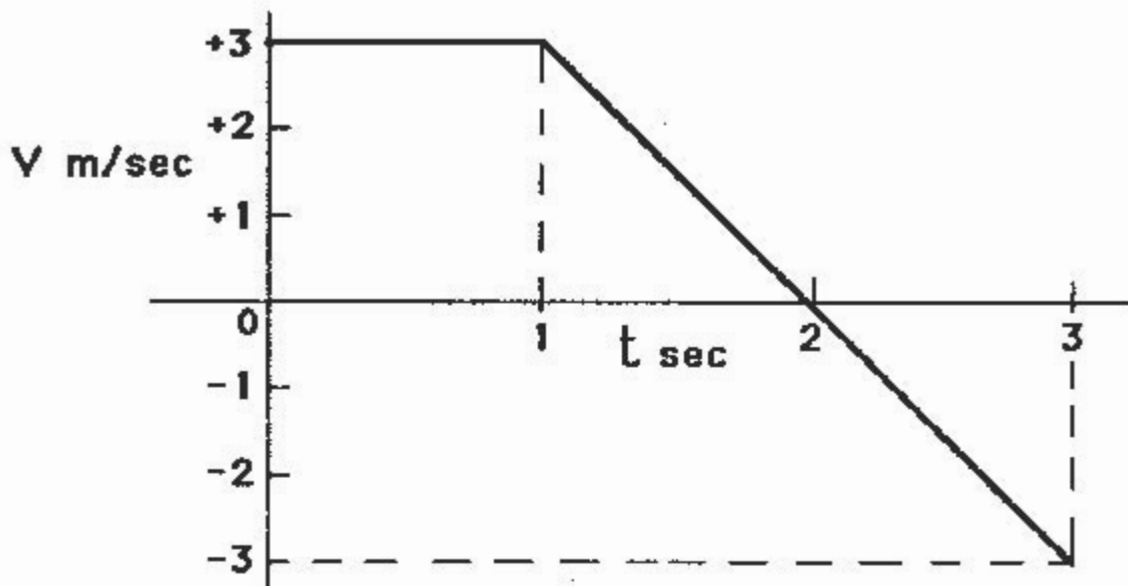
Now the particle is moving only along the z-axis, and its position is given by

$$(t^2 - 2t - 3)\hat{z}.$$

- d) (6) At what time does the particle stand still?
- e) (10) Make a plot (a sketch) of  $z$  versus time covering  $t = -2$  to  $+4$  sec.

**Problem 3 (44 points)**

A particle moves along a straight line,  $x$ . At time  $t = 0$ , its position is at  $x = 0$ . The velocity,  $V$ , of the object changes as a function of time,  $t$ , as indicated in the figure;  $t$  is in seconds,  $V$  in m/sec and  $x$  in meters.



- a) (6) What is  $x$  at  $t = 1$  sec?
- b) (6) What is the acceleration (in  $\text{m/sec}^2$ ) at  $t = 2$  sec?
- c) (6) What is  $x$  at  $t = 3$  sec?
- d) (6) What is the average velocity (in  $\text{m/sec}$ ) between  $t = 0$  and  $t = 3$  sec?
- e) (10) What is the average speed (in  $\text{m/sec}$ ) between  $t = 0$  and  $t = 3$  sec?
- f) (10) Make a plot (a sketch) of  $x$  versus time between  $t = 0$  and  $t = 3$  sec. Indicate clearly in your plot at  $t = 0, 1, 2, 3$  sec what exactly the  $x$  positions are (be quantitative).

**Problem 2** 34 points

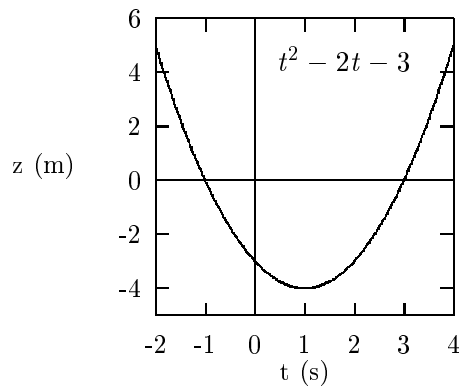
6 pts a)  $\vec{v} = \frac{d\vec{r}}{dt} = 4\hat{y} - (2 - 2t)\hat{z}$   
at  $t = 3$ ,  $\vec{v} = 4\hat{y} + 4\hat{z}$

6 pts b)  $|\vec{v}| = \sqrt{16 + 16} = 4\sqrt{2} \text{ m/sec}$

6 pts c)  $\vec{a} = 2\hat{z}$ ,  $|\vec{a}| = 2 \text{ m/sec}$

6 pts d)  $v = (2t - 2)\hat{z} \Rightarrow v = 0$  when  $t = 1 \text{ sec}$

10 pts e)  $z = t^2 - 2t - 3$ , at  $t = 0$ ,  $z = -3$   
 $z = 0 \rightarrow t = \frac{+2 \pm \sqrt{4 + 12}}{2} = 1 \pm 2 \Rightarrow t = -1$  and  $t = +3$   
 $v = 0$  at  $t = 1 \Rightarrow z = 1 - 2 - 3 = -4$   
at  $t = -2$ ,  $z = 4 + 4 - 3 = +5$



**Problem 3** 44 points

6 pts a)  $x = x_0 + v_0 t = 3t$  and at  $t = 1$ ,  $x = +3\text{ m}$

6 pts b)  $a = \frac{dv}{dt}$  and  $a$  is constant between  $t = 1$  and  $t = 3$ . The velocity goes down by 6 m/sec in 2 sec. Thus,  $a = -3 \text{ m/sec}^2$ .

6 pts c) At the beginning of the 2<sup>nd</sup> sec,  $x = +3$  and  $v = +3$ . During the next 2 sec (up to  $t = +3$ ),  $a = -3$ . Thus  $x$  at  $t = 3$  becomes  $x = +3 + 3t - \frac{3}{2}t^2$ . But  $t$  is now 2 sec so  $x = +3 \text{ m}$ .

6 pts d)  $\bar{v}_{t=0, t=3} = \frac{x_3 - x_0}{3} = \frac{+3 - 0}{3} = +1 \text{ m/sec}$

10 pts e) Between  $t = 1$  sec and  $t = 2$  sec, the position of  $x$  keeps increasing as the velocity is positive.  $x$  reaches a maximum at  $t = 2$  sec, at which time its position is  $x = +4.5 \text{ m}$ . During the third second (between  $t = 2$  sec and  $t = 3$  sec), the velocity becomes negative and at  $t = 3$  sec the object is back at  $x = +3$ . Thus, it has traveled  $4.5 + 1.5 = 6 \text{ m}$  during the first 3 sec. Thus its average speed is **2 m/sec**.

10 pts f) The plot:

