

Name: _____

Problem 3: _____ /25

Problem 3(25 points) Solve the following separable ordinary differential equation with given initial condition.

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Solution to Problem 3 Using the exponent rule,

$$e^{a+b} = e^a \cdot e^b,$$

the differential equation is equivalent to,

$$\frac{dy}{dx} = e^x \cdot e^{2y}.$$

This gives an equation of differentials,

$$e^{-2y} dy = e^x dx.$$

Antidifferentiating gives,

$$\int e^{-2y} dy = \int e^x dx,$$

which is,

$$-\frac{1}{2}e^{-2y} = e^x + C.$$

Substituting the initial condition gives the equation for C ,

$$-\frac{1}{2}e^0 = e^0 + C.$$

Solving for C gives,

$$C = \frac{-3}{2}.$$

Substituting this in and simplifying gives,

$$e^{-2y} = -2e^x + 3.$$

Taking logarithms and simplifying gives,

$$y = -\ln(3 - 2e^x)/2.$$

~~**Problem 4**(25 points) Compute each of the following Riemann integrals. You are not required to show every step. Please do say what method you use to compute the integral.~~

~~**(a)**(5 points)~~

~~$$\int_0^{\sqrt{5}} 1 + 2x + 3x^2 + 4x^3 + 5x^4 dx$$~~