

**Problem 5. a.** (10 pts) Evaluate the following indefinite integrals

i.  $\int \cos(3x) dx$

ii.  $\int x e^{(x^2)} dx$

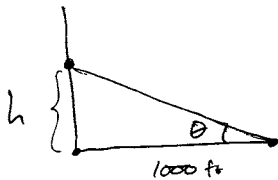
**b.** (10 pts) Find  $y(x)$  such that  $y' = \frac{1}{y^3}$  and  $y(0)=1$

**Problem 6.** (10 pts.) Suppose that  $f'(x) = e^{(x^2)}$ , and  $f(0)=10$   
One can conclude from the mean value theorem that

$$A < f(1) < B$$

for which numbers A and B?

Problem 4. (15 pts.) A rocket is launched straight up, and its altitude is  $h = 10t^2$  feet after  $t$  seconds. You are on the ground 1000 feet from the launch site. The line of sight from you to the rocket makes an angle  $\theta$  with the horizontal. By how many radians per second is  $\theta$  changing ten seconds after the launch?



$$\tan \theta = \frac{h}{1000} = \frac{10t^2}{1000} = \frac{1}{100} t^2$$

$$\sec^2 \theta \frac{d\theta}{dt} = \frac{1}{50} t$$

$$\text{when } t = 10, h(10) = 10 \cdot 10^2 = 1000, \text{ so}$$

$$\tan \theta = 1 \text{ so } \theta = \pi/4$$

$$\frac{1}{\cos^2(\pi/4)} \frac{d\theta}{dt} \Big|_{t=10} = \frac{1}{50} \cdot 10$$

$$\frac{d\theta}{dt} \Big|_{t=10} = \frac{1}{5} \cos^2 \frac{\pi}{4} = \frac{1}{5} \left( \frac{\sqrt{2}}{2} \right)^2 = \boxed{\frac{1}{10} \text{ rad/s}}$$

Problem 5. a. (10 pts) Evaluate the following indefinite integrals

$$\text{i. } \int \cos(3x) dx = \frac{1}{3} \sin(3x) + C$$

$$\text{ii. } \int x e^{x^2} dx = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C$$

$$u = x^2$$

$$du = 2x dx$$

$$= \frac{1}{2} e^{x^2} + C$$

b. (10 pts) Find  $y(x)$  such that  $y' = \frac{1}{y^3}$  and  $y(0) = 1$

$$\frac{dy}{dx} = \frac{1}{y^3}$$

$$y^3 dy = dx$$

$$\frac{1}{4} y^4 = x + C$$

$$\frac{1}{4} = C$$

$$y = (4x+1)^{1/4}$$

Problem 6. (10 pts.) Suppose that  $f'(x) = e^{e^x}$ , and  $f(0) = 10$ . One can conclude from the mean value theorem that

$$A < f(1) < B$$

for which numbers  $A$  and  $B$ ?

$$\frac{f(1) - f(0)}{1 - 0} = f'(c) \text{ for some } c, 0 < c < 1.$$

$$\text{Or } f(1) = f(0) + f'(c) = 10 + e^{e^c}, 0 < c < 1.$$

$$\text{If } 0 < c < 1, \text{ then } e^{e^c} < e \text{ and } 1 < e^{e^c}.$$

$$\text{So } f(1) = 10 + e^{e^c} < 10 + e$$

$$\text{and } f(1) = 10 + e^{e^c} > 10 + 1 = 11$$

$$\boxed{A = 11}$$

$$\boxed{B = 10 + e}$$